Inference at $*$ 2 0
 of proof for Lemma exists_functionality_wrt_iff:

1. S : Type
2. T : Type
3. P : $S \rightarrow \mathbb{P}$
4. Q : $S \rightarrow \mathbb{P}$
5. $S = T$
6. $\forall x{:}S.\ P(x) \iff Q(x)$
7. $\exists y{:}T.\ Q(y)$

$\vdash$ $\exists x{:}S.\ P(x)$
  by PERMUTE{1:n, 1:n, 2:n}

  1: . . . . .wf. . . . . NILNIL

     7. y : T
     8. $Q(y)$
     $\vdash$ $y \in S$
  2: . . . . .wf. . . . . NILNIL

     7. y : T
     8. $Q(y)$
     9. x : S
     $\vdash$ $P(x) \in \mathbb{P}$
  .